Unit - I

Defn:

The eqn $f(x)=0$ is called a (R.E) reciprocal eqn if whenever α is a root of the eqn and $\frac{1}{\alpha}$ is also a root.

Eg. 1) $x^2+2x+1=0$ 		$1 \quad 2$

$1 \cdot 1 \quad 1+1$

$(x+1)(x+1)=0$

$x=-1, \quad x=-1$

It is a R.E, since its roots are $-1, -1$.

2) $2x^2-5x+2=0$

$(x-2)(x-\frac{1}{2})=0$

$x=2, \frac{1}{2}$.

It is a R.E, since its roots are $2, \frac{1}{2}$

(3) $2x^3+3x^2-3x-2=0$.

$x=1, \quad 2x^2+5x+2=0$

$x=1, \quad (x+2)(x+\frac{1}{2})=0$

$x=1. \quad x=-2, -\frac{1}{2}$

It is a R.E, since its roots are $-2, -\frac{1}{2}, 1$.

Theorem: 1

If $\alpha_1, \alpha_2, \dots \alpha_n$ be the roots of $f(x)=0$, then $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \dots, \frac{1}{\alpha_n}$ are the roots of $x^n f(\frac{1}{x})=0$

Proof:

Since $\alpha_1, \alpha_2, \dots, \alpha_n$ are the roots of $f(x)=0$ we have $f(\alpha_i)=0$, $i=1,2,3,\dots,n$.

Let $g(x)=x^n f(\frac{1}{x})$

Then $g(\frac{1}{\alpha_i}) = (\frac{1}{\alpha_i})^n f(\alpha_i)$

$g(\frac{1}{\alpha_i})=0, \forall i=1 \text{ to } n, \because f(\alpha_i)=0$

and hence $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \dots \frac{1}{\alpha_n}$ are the roots of $g(x)$

ie) $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \dots, \frac{1}{\alpha_n}$ are the roots of $x^n f(\frac{1}{x})=0$.

Examples of R.E:

If $\alpha_1, \alpha_2, \dots \alpha_n$ are roots of Eqn $f(x)$, then $f(x)$ can be formed by

$$x^n - S_1 x^{n-1} + S_2 x^{n-2} - S_3 x^{n-3} + \dots + (-1)^n S_{n-1} x + S_n = 0$$

$S_1 = \alpha_1+\alpha_2+\dots+\alpha_n$

$S_2 = \alpha_1\alpha_2+\alpha_1\alpha_3+\dots+\alpha_1\alpha_n$

$\alpha_2\alpha_3+\alpha_2\alpha_4+\dots+\alpha_2\alpha_n$

$S_3 = \alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\dots$

$S_4 = \alpha_1\alpha_2\alpha_3\alpha_4+\dots$

$\vdots$

$S_n = \alpha_1\alpha_2\alpha_3\dots\alpha_n$

1. Roots $\alpha = 7$, $\beta = \frac{1}{7}$

$\alpha + \beta = 7 + \frac{1}{7} = \frac{50}{7}$

$\alpha\beta = 7 \times \frac{1}{7} = 1$

Required Equation :

$x^2 - \frac{50}{7}x + 1 = 0$

$7x^2 - 50x + 7 = 0$

2. $\alpha = 12$, $\beta = \frac{1}{12}$

Required Equation :

$12x^2 - 145x + 12 = 0$

3. $\alpha = 18$, $\beta = \frac{1}{18}$

$18x^2 - 325x + 1 = 0$

4. $\alpha = 8$, $\beta = \frac{1}{8}$

$8x^2 - 65x + 8 = 0$

5. $\alpha = 1$, $\beta = 1$

$x^2 - 2x + 1 = 0$

6. $\alpha = -1$, $\beta = -1$

$x^2 + 2x + 1 = 0$

7. $\alpha = 2$ $\beta = \frac{1}{2}$ $\gamma = 1$

$2x^3 - 7x^2 + 7x - 2 = 0$

8. $\alpha = 7$, $\beta = \frac{1}{7}$ $\gamma = 1$

$7x^3 - 57x^2 + 57x - 7 = 0$

9. $\alpha = 5$, $\beta = \frac{1}{5}$ $\gamma = -1$

$5x^3 - 21x^2 - 21x + 5 = 0$

10. $\alpha = 2$, $\beta = \frac{1}{2}$, $\gamma = 3$, $\delta = \frac{1}{3}$

$6x^4 - 35x^3 - 59x^2 - 35x + 6 = 0$

Remark :

If $f(x) = 0$ is a reciprocal eq. then $f(x) = 0$ and $x^n f\left(\frac{1}{x}\right)$ both have the same roots.

Eg: $7x^3 - 57x^2 + 57x - 7 = f(x)$

$x^3 f\left(\frac{1}{x}\right) = x^3 \cdot 7\left(\frac{1}{x^3}\right) - x^3 57 \frac{1}{x^2} + x^3 57\left(\frac{1}{x}\right) - x^3 7$

$= 7 - 57x + 57x^2 - 7x^3$

$= -(7x^3 - 57x^2 + 57x - 7)$

Theorem :

$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$ is a R.E iff $a_{n-r} = \pm a_r$, where $0 \le r \le n$.

Proof :

$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$

$x^n f\left(\frac{1}{x}\right) = x^n \left[a_0 \left(\frac{1}{x}\right)^n + a_1 \left(\frac{1}{x}\right)^{n-1} + \dots + a_{n-1}\left(\frac{1}{x}\right) + a_n\right]$

$= a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_0$

Now suppose $f(x) = 0$ is a R.E

Then $f(x) = 0$ and $x^n f(\frac{1}{x}) = 0$ both have the same roots and hence the corresponding coeff. of the two eqns. are proportional.

$$\frac{a_0}{a_n} = \frac{a_1}{a_{n-1}} = \cdots = \frac{a_r}{a_{n-r}} = \frac{a_n}{a_0} = k$$

Since $\frac{a_0}{a_n} = \frac{a_n}{a_0} = k$,

$$k^2 = 1$$

such that $k = \pm 1$.

$$\frac{a_r}{a_{n-r}} = \pm 1$$

and hence

$$a_{n-r} = \pm a_r.$$

Conversely suppose that

$a_{n-r} = \pm a_r$

Then the eqns. $f(x) = 0$ and $x^n f(\frac{1}{x}) = 0$ are same and hence $f(x) = 0$ is a ~~R.E~~. R.E.

Defn.

The R.E $f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n = 0$ is said to be of first type if $a_{n-r} = a_r$ and is said to be of second type if $a_{n-r} = -a_r$.

Remark:

$f(x) = 0$ is a R.E of first type iff $f(x) = x^n f(\frac{1}{x})$ and second type $f(x) = -x^n f(\frac{1}{x})$.

Standard form of R.E:

Defn:

A reciprocal eqn. $f(x) = 0$ is called a standard reciprocal equation (S.R.E) if it is of first type and the degree of $f(x)$ is even.

Eg: $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$

*) is a R.E of even degree and first type.

∴ This eqn. is S.R.E

* $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$ is a R.E of first type and odd ~~of~~ degree.

* $x^5-5x^4+9x^3-9x^2+5x-1=0$ is a R.E of second type and odd degree.

iv) $7x^4-8x^3-9x^2+8x-7=0$ is a R.E of second type and even degree.

v) $3x^7+4x^6-5x^5+6x^4+6x^3-5x^2+4x+3$ is a R.E of first type and odd degree.

vi) $4x^8-6x^7+8x^6-10x^5+12x^4-10x^3+8x^2-6x+4$ is S.R.E since it is a R.E of first type and even degree.

Result:

Any R.E can be reduced to a S.R.E on divison by a suitable factor.

* If $f(x)=0$ is a R.E of first type and odd degree, then $x+1$ is a factor of $f(x)$ and $\frac{f(x)}{x+1}$ is a S.R.E

* If $f(x)=0$ is a R.E of second type and odd degree, then $x-1$ is a factor of $f(x)$ and $\frac{f(x)}{x-1}$ is a S.R.E.

* If $f(x)=0$ is a R.E of second type and even degree, then x^2-1 is a factor of $f(x)$ and $\frac{f(x)}{x^2-1}$ is a S.R.E.

A Reciprocal eqn. of the standard form can always be depressed to another of half the dimensions.

Let $f(x)=a_0x^{2m}+a_1x^{2m-1}+\cdots+a_{2m}=0$ be a S.R.E. so that

$$a_{2m-r}=a_r.$$

$$\therefore f(x)=a_0(x^{2m}+1)+a_1(x^{2m-1}+x)+\cdots+a_{m-1}(x^{m+1}+x^{m-1})+a_m$$

$\div x^m$

$$\frac{f(x)}{x^m}=a_0\left(x^m+\frac{1}{x^m}\right)+a_1\left(x^{m-1}+\frac{1}{x^{m-1}}\right)+\cdots+a_{n-1}\left(x+\frac{1}{x}\right)+a_m$$

Putting $x+\frac{1}{x}=y$

we obtain a polynomial $g(y)$ of degree n.

Let $y_1, y_2, \ldots, y_n$ be the roots of the eqn. $g(y)=0$.

Now, $x+\frac{1}{x}=y_i$ gives

$x^2-xy_i+1=0$ and hence for each value of y_i we get two roots of the given S.R.E.

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Problems:

Eg: 1 Find the roots of the given equation

$x^5+4x^4+3x^3+3x^2+4x+1=0$

Soln:

Let $f(x) = x^5+4x^4+3x^3+3x^2+4x+1$

Here $f(x)$ is the reciprocal eqn with odd and first type (like sign)

$\therefore (x+1)$ is a factor of $f(x)$.

$x=-1$ is a root of $f(x)$

$$\begin{array}{r|rrrrrr} -1 & 1 & 4 & 3 & 3 & 4 & 1 \\ & \downarrow & -1 & -3 & 0 & -3 & -1 \\ \hline & 1 & 3 & 0 & 3 & 1 & \boxed{0} \end{array}$$

$f(x) = (x+1)\ (x^4+3x^3+0x^2+3x+1)$

To find the remaining roots of $f(x)$.

It is enough to solve the eqn:

$x^4+3x^3+0x^2+3x+1=0 \quad \text{--- (1)}$

Divide eqn (1) by x^2, we get

$x^2+3x+0+3/x+\frac{1}{x^2}=0$

$\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)=0 \quad \text{--- (2)}$

Let $y = x+\frac{1}{x}$

$y^2 = \left(x+\frac{1}{x}\right)^2$

$x^2+\frac{1}{x^2}+2 = y^2$

$x^2+\frac{1}{x^2} = y^2-2$

Then eqn (2) becomes,

$y^2-2+3y=0 \Rightarrow y^2+3y-2=0$

$y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$y = \frac{-3\pm\sqrt{17}}{2}$

$x+\frac{1}{x} = \frac{-3+\sqrt{17}}{2}, \quad x+\frac{1}{x} = \frac{-3-\sqrt{17}}{2}$

$2(x^2+1) = (-3+\sqrt{17})x; \quad 2(x^2+1) = (-3-\sqrt{17})x$

$2x^2+3x-\sqrt{17}x+2=0,$

$2x^2+3x+\sqrt{17}x+2=0.$

These two eqn yields the required outs of the given eqn.

Eg: Solve the eqn.

$6x^5-x^4-43x^3+43x^2+x-6=0$

Soln.

Let $f(x) = 6x^5-x^4-43x^3+43x^2+x-6=0$

Here, $f(x)$ is a reciprocal eqn of odd degree with unlike sign.

$\Rightarrow (x-1)$ is a factor of $f(x)$.

$x=1$ is a root of $f(x)$

$$\begin{array}{r|rrrrrr} 1 & 6 & -1 & -43 & 43 & 1 & -6 \\ & 0 & 6 & 5 & -38 & 5 & 6 \\ \hline & 6 & 5 & -38 & 5 & 6 & \boxed{0} \end{array}$$

$f(x) = (x-1)(6x^4+5x^3-38x^2+5x+6)$

To find the remaining root of $f(x)$.

It is enough to solve the eqn.

$$6x^4+5x^3-38x^2+5x+6 = 0 \quad \text{—(1)}$$

Divide equation (1) by x^2

$$6x^2+5x-38+5/x+6/x^2 = 0$$

$$6\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)-38 = 0 \quad \text{—(2)}$$

Put $x+\frac{1}{x} = y \Rightarrow \left(x+\frac{1}{x}\right)^2 = y^2$

$$x^2+\frac{1}{x^2} = y^2-2$$

$\therefore$ eqn (2) becomes

$$6y^2-12+5y-38 = 0$$

$$6y^2+5y-50 = 0$$

$$6y^2+20y-15y-50 = 0$$

$$2y(3y+10)-5(3y+10) = 0$$

$3y+10 = 0$ $\qquad$ $2y-5 = 0$

$y = \frac{-10}{3}$ $\qquad$ $y = 5/2$

$x+\frac{1}{x} = \frac{-10}{3}$ $\qquad$ $x+\frac{1}{x} = 5/2$

$3(x^2+1) = -10x$ $\qquad$ $2(x^2+1) = 5x$

$3x^2+10x+3 = 0$, $\qquad$ $2x^2-5x+2=0$

× 9, 10: 9, 1 → $\frac{9}{3} = 3$, $\frac{1}{3}$

4, -5: $\frac{-1}{2}$, $\frac{-4}{2}$

$$(x+3)\left(x+\frac{1}{3}\right) = 0$$

$$\left(x-\frac{1}{2}\right)\left(x-\frac{4}{2}\right) = 0$$

$x+3 = 0$, $x+\frac{1}{3} = 0$, $x-\frac{1}{2} = 0$, $x-2=$

$x = -3$, $x = \frac{-1}{3}$, $x = \frac{1}{2}$, $x = 2$.

$\therefore$ The roots of $f(x)$ are,

$$x = 1, 2, \tfrac{1}{2}, -3, -\tfrac{1}{3}.$$

Eg: 3 Solve the eqn:

$$6x^6-35x^5+56x^4-56x^2+35x-6=0$$

soln

Let $f(x) = 6x^6-35x^5+56x^4+0x^3-56x^2+35x-6 = 0$.

Here $f(x)$ is a R.E of even degree

$\therefore$ (Unlike signs).

$\Rightarrow x^2-1$ is a factor of $f(x)$.

$$x^2-1 = 0 \Rightarrow x^2 = 1$$

$$x = \pm 1$$

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$$
\begin{array}{r|rrrrrrr}
 & 6 & -35 & 56 & 0 & -56 & 35 & -6 \\
1 & 0 & 6 & -29 & 27 & 27 & -29 & 6 \\
\hline
 & 6 & -29 & 27 & 27 & -29 & 6 & \boxed{0} \\
-1 & 0 & -6 & 35 & -62 & 35 & -6 & \\
\hline
 & 6 & -35 & 62 & -35 & 6 & \boxed{0} &
\end{array}
$$

$\therefore f(x) = (x^2-1)(6x^4 - 35x^3 + 62x^2 - 35x + 6)$

It is enough to solve this eqn:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0 \rightarrow (1)$

Dividing by x^2 on eqn (1), we get

$6x^2 - 35x + 62 - 35/x + 6/x^2 = 0$

$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0 \rightarrow (2)$

Put $y = x + 1/x$

$y^2 - 2 = x^2 + \frac{1}{x^2}$

(2) $\Rightarrow$ $6(y^2-2) - 35y + 62 = 0$

$6y^2 - 12 - 35y + 62 = 0$

$6y^2 - 35y + 50 = 0$

300, -35; $\frac{-20}{6} = \frac{-10}{3}$, $\frac{-15}{6}$ $\quad (6y^2 - 20y - 15y + 50 = 0)$

$\left(y - \frac{10}{3}\right)\left(y - \frac{5}{2}\right) = 0$

$y = \frac{10}{3}$ $\qquad y = \frac{5}{2}$

$x + \frac{1}{x} = \frac{10}{3}$ $\qquad x + \frac{1}{x} = \frac{5}{2}$

$3(x^2+1) = 10x$, $2(x^2+1) = 5x$

$3x^2 + 3 = 10x$, $2x^2 + 2 = 5x$

$3x^2 - 10x + 3 = 0$, $2x^2 - 5x + 2 = 0$

9, -10; $\frac{-9}{3} = -3$, $\frac{-1}{3}$ $\qquad$ 4, -5; $\frac{-4}{2} = -2$, $\frac{-1}{2}$

$(x-3)\left(x - \frac{1}{3}\right) = 0$ $\qquad (x-2)\left(x - \frac{1}{2}\right) = 0$

$x = 3, \frac{1}{3}$ $\qquad x = 2, \frac{1}{2}$

The roots of $f(x)$ are

$x = 1, -1, 3, \frac{1}{3}, 2, \frac{1}{2}$.

H.W:

1. Solve the following eqns:

1) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

2) $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$

$x^5 + x^4 + x^3 + x^2 + x + 1 = 0$ $\quad -1, \frac{\pm 1 \pm \sqrt{-3}}{2}$

$6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$

$x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$

$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

Ans: 1. $3+\sqrt{8}, 3-\sqrt{8}, 2+\sqrt{3}, 2-\sqrt{3}$

2. $\frac{1}{2}, 2, \frac{1}{2}, 2$

3. $x^4 + 3x^3 - 3x - 1 = 0$

Ans: $1, -1, \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$

4. $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$

Ans: $2, \frac{1}{2}, \frac{3+\sqrt{5}}{2}, 3-\sqrt{5}/2, \frac{-1+i\sqrt{3}}{2}, \frac{-1-\sqrt{-3}}{2}$

To increase or decrease the roots of a given eqn by a given quantity:

* Let $\alpha_1, \alpha_2, \dots \alpha_n$ are be the roots of n^{th} degree equation $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$, to form an equation whose roots are decreased by h.

i.e) $\alpha_1 - h, \alpha_2 - h, \dots \alpha_n - h$.

* Let $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$.
* Let $y = \alpha_1 - h, \alpha_2 - h, \dots \alpha_n - h$

Now, $y = \alpha_1 - h$

$y = x - h$ $(\because x = \alpha_1$ is a root$)$

$\Rightarrow x = y + h$.

* $|||^{ly}$ Proceeding like this, we get

$a_0 (y+h)^n + a_1 (y+h)^{n-1} + \dots + a_n = 0$ —①

* Expanding and collecting the coefficients of the powers of y, let the eqn. be

$$A_0 y^n + A_1 y^{n-1} + A_2 y^{n-2} + \dots + A_{n-1} y + A_n = 0 \quad —②$$

where $A_0, A_1, A_2, \dots A_n$ are the fun^s of $a_0, a_1, \dots a_n$.

Now, $y = x - h$

$$\Rightarrow A_0 (x-h)^n + A_1 (x-h)^{n-1} + \dots + A_n = 0 \quad —③$$

Equating ① & ③ as they are identical.

* we can easily find coefficient $A_0, A_1, \dots, A_n$.
* By dividing it by $(x-h)$,

Remainder $= A_n$

Quotient $= A_0 (x-h)^{n-1} + A_1 (x-h)^{n-2} + \dots$

If the quotient again is divided by $x - h$, the remainder is A_{n-1} and the quotient is

$A_0 (x-h)^{n-2} + A_1 (x-h)^{n-3} + \dots + A_{n-3}(x-h) + A_{n-2}$.

By continuing this process we can find all the coeffs of the transformed eqn ②.

Result:

1. To form an eqn. whose roots are decreased by h, we have to divide by h.

2. To form an eqn. whose roots are increased by h, we have to divide by -h.

Example: 1

Find the Quotient and the Remainder when $3x^3+8x^2+8x+12$ is divided by $x-4$.

soln.

Given equation is $3x^3+8x^2+8x+12$

To divide the eqn. by $x-4$, it is enough to do

$\Rightarrow x-4=0 \Rightarrow x=4$

4	3	8	8	12
	0	12	80	352
	3	20	88	$\boxed{364}$

Remainder = 364

Quotient = $3x^2+20x+88$

Eg: 2 Find the quotient and remainder when $2x^6+3x^5-15x^2+2x-4$ is by $x+5$.

Soln:

The Given eqn. is

$2x^6+3x^5-15x^2+2x-4=0$

To divide the eqn. by $x+5$, it is enough to do $x+5=0$,

$x=-5$

-5	2	3	0	0	-15	2	-4
	0	-10	35	-175	875	-4300	21490
	2	-7	35	-175	860	-4298	21486

Quotient = $2x^5-7x^4+35x^3-175x^2+860x-4298$

Remainder = 21486.

Eg: 3:

Diminish the roots of the eqn. $x^4-5x^3+7x^2-4x+5=0$ by 2

Soln.

Given equation is

$x^4-5x^3+7x^2-4x+5=0$

$$
\begin{array}{c|ccccc}
2 & 1 & -5 & 7 & -4 & 5 \\
 & 0 & 2 & -6 & 2 & -4 \\
\hline
2 & 1 & -3 & 1 & -2 & \boxed{1} \\
 & 0 & 2 & -2 & -2 & \\
\hline
2 & 1 & -1 & -1 & \boxed{-4} & \\
 & 0 & 2 & 2 & & \\
\hline
2 & 1 & 1 & \boxed{1} & & \\
 & 0 & 2 & & & \\
\hline
 & 1 & \boxed{3} & & &
\end{array}
$$

The transformed eqn. is

$$x^4+3x^3+x^2-4x+1=0.$$

Eg. 4

To diminish by 3 the roots of the eqn. $x^5-4x^4+3x^3-4x+6=0$

Soln.

Gn. eqn. is

$$x^5-4x^4+3x^3-4x-6=0$$

$$
\begin{array}{c|cccccc}
3 & 1 & -4 & 3 & 0 & -4 & +6 \\
 & 0 & 3 & -3 & 0 & 0 & -12 \\
\hline
3 & 1 & -1 & 0 & 0 & -4 & \boxed{-6} \\
 & 0 & 3 & 6 & 18 & 54 & \\
\hline
3 & 1 & 2 & 6 & 18 & \boxed{50} & \\
 & 0 & 3 & 15 & 63 & & \\
\hline
3 & 1 & 5 & 21 & \boxed{81} & & \\
 & 0 & 3 & 24 & & & \\
\hline
3 & 1 & 8 & \boxed{45} & & & \\
 & 0 & 3 & & & & \\
\hline
 & 1 & \boxed{11} & & & &
\end{array}
$$

The transformed eqn. is

$$x^5+11x^4+45x^3+81x^2+50x-6=0$$

Eg. 5

Find the eqn. whose roots are the roots of

$x^4-5x^3+7x^2-17x+11$ each diminished by 2.

Soln.

Gn. eqn.

$$x^4-5x^3+7x^2-17x+11=0$$

2	1	−5	7	−17	11
	0	2	−6	2	−30
2	1	−3	1	−15	$\boxed{-19}$
	0	2	−2	−2	
2	1	−1	−1	$\boxed{-17}$	
	0	2	2		
2	1	1	$\boxed{1}$		
	0	2			
	1	$\boxed{3}$			

The transformed eqn. is

$$x^4+3x^3+x^2-17x-19=0$$

Eg. 6: Increase by 7 the roots of the eqn.

$$3x^4+7x^3-15x^2+x-2=0$$

soln.

Increasing the roots by 7 is equivalent to diminishing the roots by −7.

The given eqn. is

$$3x^4+7x^3-15x^2+x-2=0$$

−7	3	7	−15	1	−2
	0	−21	98	−581	4060
−7	3	−14	83	−580	$\boxed{4058}$
	0	−21	245	−2296	
−7	3	−35	328	$\boxed{-2876}$	
	0	−21	392		
−7	3	−56	$\boxed{720}$		
	0	−21			
	3	$\boxed{-77}$			

The transformed eqn. is

$$3x^4-77x^3+720x^2-2876x+4058=0$$

Eg. Show that the eqn. $x^4-3x^3+4x^2-2x+1=0$ can be transformed into a reciprocal eqn. by diminishing its roots by unity. Hence solve the eqn.

soln.

First to diminish the given eqn. by 1.

$$
\begin{array}{c|ccccc}
1 & 1 & -3 & 4 & -2 & 1 \\
  & 0 & 1 & -2 & 2 & 0 \\
\hline
1 & 1 & -2 & 2 & 0 & \boxed{-1} \\
  & 0 & 1 & -1 & 1 & \\
\hline
1 & 1 & -1 & 1 & \boxed{1} & \\
  & 0 & 1 & 0 & & \\
\hline
1 & 1 & 0 & \boxed{1} & & \\
  & 0 & 1 & & & \\
\hline
  & 1 & \boxed{1} & & &
\end{array}
$$

The transformed eqn is

$x^4+x^3+x^2+x+1=0$. ——— (1)

which is a R.E of even degree and like sign.

Divide by x^2 in eqn (1)

$x^2+x+1+\frac{1}{x}+\frac{1}{x^2}=0$

$\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)+1=0$ ——— (2)

Put $x+\frac{1}{x}=y$

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2$

$x^2+\frac{1}{x^2}=y^2-2$

(2) $\Rightarrow y^2-2+y+1=0$

$y^2+y-1=0$

$y=\frac{-1\pm\sqrt{1-4(1)(-1)}}{2}$

$y=\frac{-1\pm\sqrt{1+4}}{2}$

$y=\frac{-1\pm\sqrt{5}}{2}$

$y=\frac{-1+\sqrt{5}}{2}, \quad y=\frac{-1-\sqrt{5}}{2}$

$x+\frac{1}{x}=\frac{-1+\sqrt{5}}{2}, \quad x+\frac{1}{x}=\frac{-1-\sqrt{5}}{2}$

$\frac{x^2+1}{x}=\frac{-1+\sqrt{5}}{2}, \quad \frac{x^2+1}{x}=\frac{-1-\sqrt{5}}{2}$

$2(x^2+1)=(-1+\sqrt{5})x,$

$2(x^2+1)=(-1-\sqrt{5})x$

$\Rightarrow 2x^2+2+x-\sqrt{5}x=0$

$2x^2+(1-\sqrt{5})x+2=0$

$\Rightarrow 2x^2+2+x+\sqrt{5}x=0$

$2x^2+(1+\sqrt{5})x+2=0$

$x=\frac{-(1-\sqrt{5})\pm\sqrt{(1-\sqrt{5})^2-4(2)(2)}}{2\cdot 2}$

$x=\frac{-1+\sqrt{5}\pm\sqrt{1-2\sqrt{5}+5-16}}{4}$

$$\boxed{x=\frac{-1+\sqrt{5}\pm\sqrt{-2\sqrt{5}-10}}{4}}$$

$$x = \frac{-(1+\sqrt{5}) \pm \sqrt{(1+\sqrt{5})^2 - 4\cdot 2\cdot 2}}{2\cdot 2}$$

$$= \frac{-1-\sqrt{5} \pm \sqrt{1+2\sqrt{5}+5-16}}{4}$$

$$x = \frac{-1-\sqrt{5} \pm \sqrt{2\sqrt{5}-10}}{4}$$

The roots of the original eqn. are these roots increased by 1

$$x = \frac{-1+\sqrt{5} \pm \sqrt{-2\sqrt{5}-10}}{4} + 1,$$

$$x = \frac{-1-\sqrt{5} \pm \sqrt{2\sqrt{5}-10}}{4} + 1$$

$$x = \frac{-1 \cancel{+} \sqrt{5} \pm \sqrt{-2\sqrt{5}-10} + 4}{4},$$

$$x = \frac{-1-\sqrt{5} \pm \sqrt{2\sqrt{5}-10} + 4}{4}$$

$$x = \frac{\sqrt{5}+3 \pm \sqrt{-2\sqrt{5}-10}}{4},$$

$$x = \frac{-\sqrt{5}+3 \pm \sqrt{-2\sqrt{5}-10}}{4}$$

H.W. (send)

1. Find the eqn. whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 2

2. Find the eqn whose roots are the roots of $4x^5 - 2x^3 + 7x - 3 = 0$ each increased by 2.

3. Find the eqn. each of whose roots exceeds by 2 a root of the eqn. $x^3 - 4x^2 + 3x - 1 = 0$.

4. Transform $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ into another eqn. whose roots are less by 3.

5. Find the eqn. whose roots are the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ increased by 2 and hence solve the eqn.

19. Removal of terms :

* The Concept of diminishing (or) increasing the roots of an eqn. is also helpful to remove a particular term from a given eqn. such removal of term from an eqn. may result in easy soln. of the resultant eqn.. which in turn will be helpful to find the Complete soln. of the given eqn.

* Let the given eqn. be

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0 \quad \text{--- (1)}$$

* To diminish the roots of eqn. (1) by $h > 0$.

Put $y = x-h$

$x = y+h$ in eqn (1)

$$(1) \Rightarrow a_0(y+h)^n + a_1(y+h)^{n-1} + a_2(y+h)^{n-2} + \dots + a_{n-1}(y+h) + a_n = 0$$

$[\because (a+b)^n = a^n + nC_1a^{n-1}b + nC_2a^{n-2}b^2 + \dots + nC_na^0b^n.]$

$$a_0(y^n + ny^{n-1}h + a\dots + h^n) +$$
$$a_1(y^{n-1} + (n-1)y^{n-2}h + \dots + h^{n-1}) +$$
$$a_2(y^{n-2} + (n-2)y^{n-3}h + \dots + h^{n-2}) +$$
$$\dots + a_{n-1}(y+h) + a_n = 0$$

$$a_0y^n + (na_0h + a_1)y^{n-1} +$$
$$(nC_2a_0h^2 + (n-1)a_1h + a_2)y^{n-2} +$$
$$\dots + a_0h^n + a_1h^{n-1} + \dots + a_{n-1}h + a_n$$
$$= 0 \quad \text{--- (2)}$$

To remove any term in equation (2), the value of h obtained.

for example to remove the second term in eqn. (2) we get,

$$na_0h + a_1 = 0$$

$$h = \frac{-a_1}{na_0}.$$ Diminishing the roots of eqn (1) by $h = \frac{-a_1}{na_0}$ will result processing like this we get obtain the roots of the given eqn. $f(x)$.

is an eqn in which the coefficient of y^{n-1} is zero.

Eg: 1 Find the relation b/t the coefficients in the eqn. $x^4+px^3+qx^2+rx+s=0$ in order that the coefficients of x^3 and x may be removable by the same transformation.

Soln:

Let $f(x)=x^4+px^3+qx^2+rx+s=0$ —— (1)

Put $x=x+h$ in eqn (1)

$(x+h)^4+p(x+h)^3+q(x+h)^2+r(x+h)+s=0$ —— (2)

$(x+h)^4=(x+h)^2(x+h)^2$

$=x^4+4C_1x^3h+4C_2x^2h^2+4C_3xh^3+4C_4h^4$

$=x^4+4x^3h+6x^2h^2+4xh^3+h^4$

$(x+h)^3=x^3+3x^2h+3xh^2+h^3$

$(x+h)^2=x^2+2xh+h^2$.

(2) $\Rightarrow x^4+4x^3h+6x^2h^2+4xh^3+h^4+p(x^3+3x^2h+3xh^2+h^3)+q(x^2+2xh+h^2)+rx+rh+s=0$

$x^4+4x^3h+6x^2h^2+4xh^3+h^4+px^3+3px^2h+3pxh^2+ph^3+qx^2+2qxh+qh^2+rx+rh+s=0$

$x^4+(4h+p)x^3+(6h^2+3ph+q)x^2+(4h^3+3ph^2+2qh+r)x+(h^4+ph^3+rh+s)=0$

The coeffs. of x^3 and x in the transformed eqn. are zeros.

$\therefore 4h+p=0 \Rightarrow \boxed{h=-\frac{p}{4}}$

$4h^3+3ph^2+2qh+r=0$

$4\left(-\frac{p}{4}\right)^3+3p\left(-\frac{p}{4}\right)^2+2q\left(-\frac{p}{4}\right)\cancel{h}+r=0$

$-\frac{p^3}{16}+\frac{3p^3}{16}-\frac{2pqh}{4}+r=0$

$\frac{2p^3}{16}-\frac{2pqh}{4}+r=0$

$2p^3-8pqh+16r=0$

$\div 2 \quad p^3-4pq+8r=0.$

Eg: Solve the eqn. $x^4+20x^3+143x^2+430x+462=0$ by removing its second term.

Soln.

Let $f(x)=x^4+20x^3-143x^2+430x+462=0$ —— (1)

Put $x=x+h$ in eqn (1),

$(x+h)^4+20(x+h)^3+143(x+h)^2+430(x+h)+462=0$

$x^4+4C_1x^3h+4C_2x^2h^2+4C_3xh^3+4C_4h^4+20(x^3+3x^2h+3xh^2+h^3)+143(x^2+2xh+h^2)+430(x+h)+462=0$

$x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 +$
$20x^3 + 60x^2h + 60xh^2 + 20h^3 +$
$+143x^2 + 286xh + 143h^2 + 430x +$
$430h + 462 = 0$

$x^4 + (4h+20)x^3 + (6h^2+60h+143)x^2$
$+ (4h^3+60h^2+286h)x +$
$(h^4+20h^3+143h^2+430h+462)=0$ —— ①

To remove second term in eqn. ②, we got

$4h+20=0$

$4h=-20$

$\boxed{h=-5}$

Hence to remove the second term, increase the roots of the eqn. by 5.

Diminish the roots of eqn. ① by -5

$218 \times 5 = 1090$

-5	1	20	+143	430	462
	↓	-5	-75	-340	-450
-5	1	15	~~218~~ 68	90	$\boxed{12}$
	↓	-5	-50	-90	
-5	1	10	18	$\boxed{0}$	
	↓	-5	-25		
-5	1	5	$\boxed{-7}$		
	↓	-5			
	1	$\boxed{0}$			

The transformed eqn. is

~~y^4+0x~~

$x^4 + 0x^3 - 7x^2 + 0x + 12 = 0$

$x^4 - 7x^2 + 12 = 0.$

~~$x^2(x^2-7)+12=0$~~

$(x^2-3)(x^2-4)=0$

$x^2=3, \quad x^2=4$

$x=\pm\sqrt{3} \quad x=\pm 2.$

$\therefore$ The roots of the transformed eqn. are $\pm\sqrt{3}, \pm 2.$

These roots are greater than the roots of the original eqn by 5.

∴ The roots of the original eqn. are

$\sqrt{3}-5, -\sqrt{3}-5, 2-5, -2-5.$

ie) $\sqrt{3}-5, -\sqrt{3}-5, -3, -7.$

ie) $-5\pm\sqrt{3}, -3, -7.$

Exercises:

1. Remove the second term from the following equations.

1. $x^5+5x^4+3x^3+x^2+x+1=0.$

Soln.

Let $f(x) = x^5+5x^4+3x^3+x^2+x+1=0$ —— (1)

Put $x=x+h$ in eqn (1).

$(x+h)^5+5(x+h)^4+3(x+h)^3+(x+h)^2 + (x+h)+1 = 0.$

$x^5+5C_1x^4h+5C_2x^3h^2+5C_3x^2h^3+5C_4xh^4+5C_5h^5+5(x^4+4C_1x^3h+4C_2x^2h^2+4C_3xh^3+4C_4h^4)+3(x^3+3x^2h+3xh^2+h^3)+(x^2+2xh+h^2)+x+h+1=0$

$x^5+(5h+5)x^4+(10h^2+4h+6h^2+3)x^3+(10h^3+6h^2+9h+1)x^2 + \cdots = 0$ —— (2)

To remove second term in eqn. (2), we get.

$5h+5=0$

$5h=-5$

$\boxed{h=-1}$

Hence to remove the second term, increase the roots of the eqn. by 1.

Diminish the roots of the eqn. (1) by -1.

-1	1	5	3	1	1	1
	↓	-1	-4	1	-2	1
-1	1	4	-1	2	-1	$\boxed{2}$
	↓	-1	-3	4	-6	
-1	1	3	-4	6	$\boxed{-7}$	
	↓	-1	-2	6		
-1	1	2	-6	$\boxed{12}$		
	↓	-1	-1			
-1	1	1	$\boxed{-7}$			
	↓	-1				
	1	$\boxed{0}$				

The transformed eqn. is

$x^5-7x^3+12x^2-7x+2=0$

2) $x^3-6x^2+10x-3=0$

Soln.

Let $f(x)=x^3-6x^2+10x-3=0$ — (1)

Put $x=x+h$ in eqn (1).

$(x+h)^3-6(x+h)^2-10(x+h)-3=0$

$x^3+3x^2h+3xh^2+h^3-6(x^2+2xh+h^2)$
$-10x-10h-3=0$

$x^3+(3h-6)x^2+(3h^2-12h-10)x+$
$(h^3-6h^2-10h-3)=0$ — (2)

To remove second term in eqn (2) we get

$3h-6=0$

$h=6/3=2$

$\boxed{h=2}$

Diminish the roots of the eqn (1) by 2

2	1	-6	10	-3
	↓	2	-8	4
	1	-4	2	$\boxed{1}$
2	↓	2	-4	
	1	-2	$\boxed{-2}$	
2	↓	2		
	1	$\boxed{0}$		

The transformed eqn is

$x^3-2x+1=0$

2) Transform the eqn $x^4-4x^3-18x^2-3x+2=0$ into one which shall want the third term.

Soln.

Let $f(x)=x^4-4x^3-18x^2-3x+2=0$ — (1)

Put $x=x+h$ in eqn (1).

$(x+h)^4-4(x+h)^3-18(x+h)^2-$
$3(x+h)+2=0$

$x^4+4x^3h+6x^2h^2+4xh^3+h^4-$
$4(x^3+3x^2h+3xh^2+h^3)-$
$18(x^2+2xh+h^2)-3x-3h+2=0$

$x^4+(4h-4)x^3+(6h^2-12h-18)x^2$
$+(4h^3-12h^2-36h-3)x+$
$(h^4-4h^3-18h^2-3h+2)=0$ — (2)

To remove the ~~second~~ third term in eqn (2)

$6h^2-12h-18=0$

$\div 6 \quad h^2-2h-3=0$

$(h-3)(h+1)=0$

$h=3, \quad h=-1$

Case (i): $h=3$.

We diminish the root of eqn (1) by 3 as follows.

3	1	-4	-18	-3	2
	↓	3	-3	-63	-198
3	1	-1	-21	-66	$\boxed{-196}$
	↓	3	6	-45	
3	1	2	-15	$\boxed{-111}$	
	↓	3	15		
3	1	5	$\boxed{0}$		
	↓	3			
	1	$\boxed{8}$			

The transformed eqn is

$y^4+8y^3-111y-196=0$

Case (ii): $h=-1$,

we diminish the roots of eqn (1) by -1, as follows.

-1	1	-4	-18	-3	2
	↓	-1	5	13	-10
-1	1	-5	-13	10	$\boxed{-8}$
	↓	-1	6	+7	
-1	1	-6	-7	$\boxed{17}$	
	↓	-1	7		
-1	1	-7	$\boxed{0}$		
	↓	-1			
	1	$\boxed{-8}$			

The transformed eqn is

$y^4-8y^3+17y-8=0$.

H.W:

1. Remove second term from the eqn. $\rightarrow h=3$

$5, 1, \sqrt{2}+3, -\sqrt{2}+3$

i) $x^4-12x^3+48x^2-72x+35=0$

ii) $x^4+16x^3+83x^2+152x+84=0$

iii) $x^3+6x^2+12x-19=0$.

Hornor's Method

is used finding approximate values of the irrational roots of the eqn $f(x)=0$, $f(x)$ is a poly.

Procedure:

Hornor's method is used to determine a real root of a numerical polynomial eqn $f(x)=0$, correct to given place of decimal.

ie) The roots $a.\alpha_1 d_2 d_3 \ldots$

Step 1:

* We are going to find the integral part a by trial find 2 consecutive integers where a real positive roots of the given eqn lies.
* Let a and a+1 be consecutive integers, such that $f(a)$ and $f(a+1)$ are opposite signs.

therefore a root lies b/t a and a+1.

~~step 2~~:

Therefore the integral part of root is a. Let the root be

$a.d_1,d_2,d_3,\ldots$

Step 2:

To find d_1

To diminish the roots of the eqn by a. Now, eqn is

$\phi_1(x)=0$ —— ① will have roots b/t zero and one.

multiply the roots of eqn ① by 10.

ie) The coeffs of $\phi_1(x)$ are multiplied by 1, 10, 100, 1000, ... respectively.

by trial find the integer b/t which the roots of eqn ① lies which is d_1. Now eqn will be

$\phi_2(x)=0$.

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Eg: 1

The eqn. $x^3-3x+1=0$ has a root b/t 1 and 2. Calculate it to three places of decimals.

Soln.

Step 1: Gn. eqn. is $x^3-3x+1=0$.

Let $f(x)=x^3-3x+1$.

$f(1)=1-3+1=-1<0$ (−ve)

$f(2)=2^3-3(2)+1$

$=8-6+1=3>0$ (+ve)

$\therefore$ A root lies b/t 1 and 2.

Let the root be $1.d_1d_2d_3\ldots$

Step 2: To find d_1

Diminish the root of $f(x)=0$ by 1

$$\begin{array}{c|cccc}
1 & 1 & 0 & -3 & 1 \\
 & \downarrow & 1 & 1 & -2 \\
\hline
1 & 1 & 1 & -2 & \boxed{-1} \\
 & \downarrow & 1 & 2 & \\
\hline
1 & 1 & 2 & \boxed{0} & \\
 & \downarrow & 1 & & \\
\hline
 & 1 & \boxed{3} & &
\end{array}$$

The transformed eqn. is

$f_1(x)=x^3+3x^2-1$.

Step 3:

Multiply the roots of the transformed eqn $f_1(x)$ by 10.

Therefore the transformed eqn. is

$f_2(x)=x^3+30x^2-1000=0$

Now

$f_2(1)=1^3+30(1)^2-1000=-969<0$

$f_2(2)=8+30(4)-1000=$

$f_2(3)=27+30(9)-1000=$

$f_2(4)=64+30(16)-1000=$

$f_2(5)=125+30(25)-1000=$ <0

$f_2(6)=216+30(36)-1000=$ >0

$\therefore$ The root of $f_2(x)$ lies b/t 5 and 6.

Let the root be $1.5d_2d_3\ldots$

Step 4: To find d_2,

diminish the transformed eqn $f_2(x)$ by 5

$$\begin{array}{c|cccc}
5 & 1 & 30 & 0 & -1000 \\
 & \downarrow & 5 & 175 & 875 \\
\hline
5 & 1 & 35 & 175 & \boxed{-125} \\
 & \downarrow & 5 & 200 & \\
\hline
5 & 1 & 40 & \boxed{375} & \\
 & \downarrow & 5 & & \\
\hline
 & 1 & \boxed{45} & &
\end{array}$$

The transformed eqn is

$f_3(x)=x^3+45x^2+375x-125=0$

Step 5:

Multiply the roots of the transformed eqn $f_3(x)$ by 10

The transformed eqn is

$f_4(x) = x^3 + 450x^2 + 37500x - 125000$

$f_4(0) = -125000 < 0 \quad (-ve)$

$f_4(1) = 1 + 450 + 37500 - 125000 < 0 \; (-ve)$

$f_4(2) = 8 + 450(4) + 37500(2) - 125000$

$= \quad < 0$

$f_4(3) = \quad < 0$

$f_4(4) = \quad > 0$

$\therefore$ The root lies b/t 3 and 4.

Let the root be $1.53d_3$

Step 6: To find d_3.

diminish the root by 3.

3	1	450	37500	-125000
	↓	3	1359	116577
3	1	453	38859	-8423
	↓	3	1368	
3	1	456	40227	
	↓	3		
	1	459		

The transformed eqn is

$f_5(x) = x^3 + 459x^2 + 40227x - 8423 = 0$

Step 6:

Multiply the roots of the transformed eqn $f_5(x)$ by 10

$f_6(x) = x^3 + 4590x^2 + 4022700x - 8423000 = 0$

$f_6(0) = -8423000 < 0 \quad (-ve)$

$f_6(1) = \quad < 0 \quad (-ve)$

$f_6(2) = \quad < 0 \quad (-ve)$

$f_6(3) = \quad > 0 \quad (+ve)$

$\therefore$ The root lies b/t 2 and 3.

The root is 1.532.

Step 7: Diminish the root by 2

2	1	4590	4022700	-8423000
	↓	2	9184	8063768
2	1	4592	4031884	-359232
	↓	2	9188	
2	1	4594	4041072	
	↓	2		
	1	4596		

$x^3 + 4596x^2 + 4041072x - 359232 = 0$

Multiply the roots by 10

$x^3 + 45960x^2 + 404107200x - 359230000$

$d_4 = -\left(\frac{-35923000}{404107200}\right) = 0.0889$

2) Find the positive root of the eqn: $x^3-2x^2-3x-4=0$. Correct to three places of decimals.

soln:

Step 1: Gn. eqn is $x^3-2x^2-3x-4=0$

Let $f(x)=x^3-2x^2-3x-4=0$

$f(1)=1-2-3-4=-8 \quad <0$ (-ve)

$f(2)=8-8-6-4=-10 \quad <0$ (-ve)

$f(3)=27-18-9-4=-4 \quad <0$ (-ve)

$f(4)=64-32-12-4=16 \quad >0$ (+ve)

∴ The root lies b/t 3 & 4.

Lett the roots be $3.d_1d_2d_3$..

Step 2: To find d_1,

diminish by 3

$$\begin{array}{c|cccc}
3 & 1 & -2 & -3 & -4 \\
 & \downarrow & 3 & 3 & 0 \\
\hline
3 & 1 & 1 & 0 & \boxed{-4} \\
 & \downarrow & 3 & 12 & \\
\hline
3 & 1 & 4 & \boxed{12} & \\
 & \downarrow & 3 & & \\
\hline
 & 1 & \boxed{7} & &
\end{array}$$

The transformed eqn is

$f_1(x)=x^3+7x^2+12x-4=0$.

Step 3:

Multiply the roots of the transformed eqn $f_1(x)$ by 10.

$f_2(x)=x^3+70x^2+1200x-4000=0$

$f_2(0)=-4000 \quad <0$

$f_2(1)=1+70+1200-4000 \quad <0$

$f_2(2)=8+280+2400-4000 \quad <0$

$f_2(3)=27+560+3600-4000 \quad >0$

∴ The root lies b/t 2 and 3.

The root is $3.2d_2d_3$.

Step 4: To find d_2,

diminishing the root ~~the~~ by 2

$$\begin{array}{c|cccc}
2 & 1 & 70 & 1200 & -4000 \\
 & \downarrow & 2 & 144 & 2688 \\
\hline
2 & 1 & 72 & 1344 & \boxed{-1312} \\
 & \downarrow & 2 & 148 & \\
\hline
2 & 1 & 74 & \boxed{1492} & \\
 & \downarrow & 2 & & \\
\hline
 & 1 & \boxed{76} & &
\end{array}$$

The transformed eqn is

$f_3(x)=x^3+76x^2+1492x-1312=0$

Step 5: Multiply the roots of the transformed eqn $f_3(x)$ by 10.

$f_4(x)=x^3+760x^2+149200x-1312000=0$

$f_4(1) = \quad < 0$

$f_4(2) = \quad < 0$

$f_4(3) = \quad < 0$

$f_4(4) = \quad < 0$

$f_4(5) = \quad < 0$

$f_4(6) = \quad < 0$

$f_4(7) = \quad < 0$

$f_4(8) = \quad < 0$

$f_4(9) = \quad > 0$

$\therefore$ The root lies b/t 8 and 9.

$\therefore$ The root is $3.28d_3$

Step : 6 To find d_3, diminish the roots by 8

$$
\begin{array}{c|cccc}
8 & 1 & 760 & 149200 & -1312000 \\
 & \downarrow & 8 & 6144 & 1242752 \\
\hline
8 & 1 & 768 & 155344 & \boxed{-69248} \\
 & \downarrow & 8 & 6208 & \\
\hline
8 & 1 & 776 & \boxed{161552} & \\
 & \downarrow & 8 & & \\
\hline
 & 1 & \boxed{784} & &
\end{array}
$$

The transformed eqn is

$f_5(x) = x^3 + 784x^2 + 161552x - 69248$

Step 7:

Multiply the roots of the transformed eqn $f_5(x)$ by 10

$f_6(x) = x^3 + 7840x^2 + 16155200x - 69248000.$

$f_6(0) = \quad -ve$

$f_6(1) =$

$f_6(2) =$

$f_6(3) =$

$f_6(4) = \quad -ve$

$f_6(5) = \quad +ve$

$\therefore$ The root lies b/t 4 and 5.

The root is 3.284

H.W:

1. Find the root b/t 0 and 1 correct to three places of decimals of the eqn. $x^3+18x-6=0$.

2) Find the root of the eqn. $x^3+5x-11=0$ which lies b/t 2 and 3 correct to two places of decimals.