

# Transportation Problem and Assignment problem

**Transportation problem** is a special kind of **Linear Programming Problem (LPP)** in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem.

## Types of Transportation problems:

**Balanced:** When both supplies and demands are equal then the problem is said to be a balanced transportation problem.

**Unbalanced:** When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

## Methods to Solve:

To find the initial basic feasible solution there are three methods:

1. NorthWest Corner Cell Method.
2. Least Call Cell Method.
3. Vogel's Approximation Method (VAM).

## Basic structure of transportation problem:

		Destination				
		D1	D2	D3	D4	Supply( $s_i$ )
Source	O1	$C_{11}$	$C_{12}$	$C_{13}$	$C_{14}$	$S_1$
	O2	$C_{21}$	$C_{22}$	$C_{23}$	$C_{24}$	$S_2$
	O3	$C_{31}$	$C_{32}$	$C_{33}$	$C_{34}$	$S_3$
	O4	$C_{41}$	$C_{42}$	$C_{43}$	$C_{44}$	$S_4$
Demand( $d_j$ ):		$d_1$	$d_2$	$d_3$	$d_4$	

In the above table **D1**, **D2**, **D3** and **D4** are the destinations where the products/goods are to be delivered from different sources **S1**, **S2**, **S3** and **S4**.  $S_i$  is the supply from the source  $O_i$ .  $d_j$  is the demand of the destination  $D_j$ .  $C_{ij}$  is the cost when the product is delivered from source  $S_i$  to destination  $D_j$ .

**a) Transportation Problem : (NorthWest Corner Method)**

An introduction to Transportation problem has been discussed in the previous Section, in this, finding the initial basic feasible solution using the NorthWest Corner Cell Method will be discussed.

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

**Explanation:** Given three sources **O1**, **O2** and **O3** and four destinations **D1**, **D2**, **D3** and **D4**. For the sources **O1**, **O2** and **O3**, the supply is **300**, **400** and **500** respectively.

The destinations **D1**, **D2**, **D3** and **D4** have demands **250**, **350**, **400** and **200** respectively.

**Solution:** According to North West Corner method, (**O1**, **D1**) has to be the starting point i.e. the north-west corner of the table. Each and every value in the cell is considered as the cost per transportation. Compare the demand for column **D1** and supply from the source **O1** and allocate the minimum of two to the cell (**O1**, **D1**) as shown in the figure.

The demand for Column **D1** is completed so the entire column **D1** will be canceled. The supply from the source **O1** remains  $300 - 250 = 50$ .

Now from the remaining table i.e. excluding column **D1**, check the north-west corner i.e. (**O1**, **D2**) and allocate the minimum among the supply for the respective column and the rows. The supply from **O1** is **50** which is less than the demand for **D2** (i.e. 350), so allocate **50** to the cell (**O1**, **D2**).

Since the supply from row **O1** is completed cancel the row **O1**. The demand for column **D2** remain  $350 - 50 = 50$ .

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	250	50			<del>300</del> 0
	O2					400
	O3					500
Demand:		<del>250</del> 0	<del>350</del> 300	400	200	1200

From the remaining table the north-west corner cell is **(O2, D2)**. The minimum among the supply from source **O2** (i.e 400) and demand for column **D2** (i.e 300) is **300**, so allocate **300** to the cell **(O2, D2)**. The demand for the column **D2** is completed so cancel the column and the remaining supply from source **O2** is  $400 - 300 = 100$ .

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	250	50			<del>300</del> 0
	O2		300			<del>400</del> 100
	O3					500
Demand:		<del>250</del> 0	<del>350</del> 0	400	200	1200

Now from remaining table find the north-west corner i.e. **(O2, D3)** and compare the **O2**supply (i.e. 100) and the demand for **D2** (i.e. 400) and allocate the smaller (i.e. 100) to the cell **(O2, D2)**. The supply from **O2** is completed so cancel the row **O2**. The remaining demand for column **D3** remains  $400 - 100 = 300$ .

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	<del>250</del>	<del>50</del>			<del>300</del> 50
		3		7	4	0
	O2		<del>300</del>	<del>100</del>		<del>400</del> 100
	2	6	5	9	0	
O3						500
	8	3	3	2		
Demand:		<del>250</del> 0	<del>350</del> 300 0	<del>400</del> 300	200	1200

Proceeding in the same way, the final values of the cells will be:

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	<del>250</del>	<del>50</del>			<del>300</del> 50
		3		7	4	0
	O2		<del>300</del>	<del>100</del>		<del>400</del> 100
	2	6	5	9	0	
O3						<del>500</del> 200
	8	3	3	2	0	
Demand:		<del>250</del> 0	<del>350</del> 300 0	<del>400</del> 300 0	<del>200</del> 0	1200

**Note:** In the last remaining cell the demand for the respective columns and rows are equal which was cell (O3, D4). In this case, the supply from O3 and the demand for D4 was 200 which was allocated to this cell. At last, nothing remained for any row or column.

Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e.  $(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400$

**b) Transportation Problem: (Least Cost Cell Method)**

The **North-West Corner** method has been discussed in the previous session. In this session, the **Least Cost Cell** method will be discussed.

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

**Solution:** According to the Least Cost Cell method, the least cost among all the cells in the table has to be found which is **1** (i.e. cell **(O1, D2)**).

Now check the supply from the row **O1** and demand for column **D2** and allocate the smaller value to the cell. The smaller value is **300** so allocate this to the cell. The supply from **O1** is completed so cancel this row and the remaining demand for the column **D2** is  $350 - 300 = 50$ .

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	300	7	4	<del>300</del> 0
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	<del>350</del> 50	400	200	1200

Now find the cell with the least cost among the remaining cells. There are two cells with the least cost i.e. **(O2, D1)** and **(O3, D4)** with cost **2**. Lets select **(O2, D1)**. Now find the demand and supply for the respective cell and allocate the minimum among them to the cell and cancel the row or column whose supply or demand becomes **0** after allocation.

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

Now the cell with the least cost is **(O3, D4)** with cost **2**. Allocate this cell with **200** as the demand is smaller than the supply. So the column gets canceled.

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

There are two cells among the unallocated cells that have the least cost. Choose any at random say **(O3, D2)**. Allocate this cell with a minimum among the supply from the respective row and the demand of the respective column. Cancel the row or column with zero value.

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

Now the cell with the least cost is **(O3, D3)**. Allocate the minimum of supply and demand and cancel the row or column with zero value.

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	2	3	2	500
Demand:		250	350	400	200	1200
		0	50	150	0	

The only remaining cell is (O2, D3) with cost 5 and its supply is 150 and demand is 150 i.e. demand and supply both are equal. Allocate it to this cell.

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	2	3	2	500
Demand:		250	350	400	200	1200
		0	0	150	0	

Now just multiply the cost of the cell with their respective allocated values and add all of them to get the basic solution i.e.  $(300 * 1) + (25 * 2) + (150 * 5) + (50 * 3) + (250 * 3) + (200 * 2) = 2400$

**c) Transportation Problem: (Vogel's Approximation Method)**

The **North-West Corner** method and the **Least Cost Cell** method has been discussed in the previous session. In this session, the **Vogel's Approximation** method will be discussed.

		Destination				Supply
		D1	D2	D3	D4	
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

**Solution:**

- For each row find the least value and then the second least value and take the absolute difference of these two least values and write it in the corresponding row difference as shown in the image below. In row **O1**, **1** is the least value and **3** is the second least value and their absolute difference is **2**. Similarly, for row **O2** and **O3**, the absolute differences are **3** and **1** respectively.
- For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference as shown in the figure. In column **D1**, **2** is the least value and **3** is the second least value and their absolute difference is **1**. Similarly, for column **D2**, **D3** and **D4**, the absolute differences are **2**, **2** and **2** respectively.

		Destination				Supply	Row Difference
		D1	D2	D3	D4		
Source	O1	3	1	7	4	300	<b>2</b>
	O2	2	6	5	9	400	<b>3</b>
	O3	8	3	3	2	500	<b>1</b>
Demand:		250	350	400	200	1200	
Column Difference:		<b>1</b>	<b>2</b>	<b>2</b>	<b>2</b>		

- These value of row difference and column difference are also called as penalty. Now select the maximum penalty. The maximum penalty is 3 i.e. row O2. Now find the cell with the least cost in row O2 and allocate the minimum among the supply of the respective row and the demand of the respective column. Demand is smaller than the supply so allocate the column's demand i.e. 250 to the cell. Then cancel the column D1.

		Destination					
		D1	D2	D3	D4	Supply	Row Difference
Source	O1	3	1	7	4	300	2
	O2	2	6	5	9	<del>400</del> 150	3
	O3	8	3	3	2	500	1
	Demand:	<del>250</del> 0	350	400	200	1200	
	Column Difference:	1	2	2	2		

- From the remaining cells, find out the row difference and column difference.

		Destination					
		D1	D2	D3	D4	Supply	Row Difference
Source	O1	3	1	7	4	300	2
	O2	2	6	5	9	<del>400</del> 150	3
	O3	8	3	3	2	500	1
	Demand:	<del>250</del> 0	350	400	200	1200	
	Column Difference:	1	2	2	2		
		-	2	2	2		

- Again select the maximum penalty which is 3 corresponding to row O1. The least-cost cell in row O1 is (O1, D2) with cost 1. Allocate the minimum among supply and demand from the

respective row and column to the cell. Cancel the row or column with zero value.

		Destination				Supply	Row Difference		
		D1	D2	D3	D4				
Source	O1	<del>2</del>	300	1	7	4	<del>300</del> 0	2	3
	O2	250		6	5	9	<del>400</del> 150	3	1
	O3	8	3	3	2	2	500	1	1
Demand:		<del>250</del> 0	<del>350</del> 50	400	200	1200			
Column Difference:		1	2	2	2				
		-	2	2	2				

- Now find the row difference and column difference from the remaining cells.

		Destination				Supply	Row Difference			
		D1	D2	D3	D4					
Source	O1	<del>2</del>	300	1	7	4	<del>300</del> 0	2	3	-
	O2	250		6	5	9	<del>400</del> 150	3	1	1
	O3	8	3	3	2	2	500	1	1	1
Demand:		<del>250</del> 0	<del>350</del> 50	400	200	1200				
Column Difference:		1	2	2	2					
		-	2	2	2					
		-	3	2	7					

- Now select the maximum penalty which is 7 corresponding to column D4. The least cost cell in column D4 is (O3, D4) with cost 2. The demand is smaller than the supply for cell (O3, D4). Allocate 200 to the cell and cancel the column.

		Destination				Supply	Row Difference		
		D1	D2	D3	D4				
Source	O1	<del>2</del>	<del>300</del>	1	7	<del>300</del> 0	2	3	-
	O2	<del>250</del>		6	5	400 150	3	1	1
	O3	8	3	3	<del>200</del>	<del>500</del> 300	1	1	1
Demand:		<del>250</del>	<del>350</del>	400	<del>200</del>	1200			
Column Difference:		1	2	2	2				
		-	2	2	2				
		-	3	2	7				

- Find the row difference and the column difference from the remaining cells.

		Destination				Supply	Row Difference			
		D1	D2	D3	D4					
Source	O1	<del>2</del>	<del>300</del>			<del>300</del> 0	2	3	-	-
	O2	<del>250</del>				400 150	3	1	1	1
	O3	8	3		<del>200</del>	<del>500</del> 300	1	1	1	0
Demand:		<del>250</del>	<del>350</del>	400	<del>200</del>	1200				
Column Difference:		1	2	2	2					
		-	2	2	2					
		-	3	2	7					
		-	3	2	-					

- Now the maximum penalty is 3 corresponding to the column D2. The cell with the least value in D2 is (O3, D2). Allocate the minimum of supply and demand and cancel the column.

		Destination				Supply	Row Difference			
		D1	D2	D3	D4					
Source	O1	<del>2</del>	<del>300</del>	<del>1</del>	<del>7</del>	<del>300</del> 0	2	3	-	-
	O2	<del>250</del>				400 150	3	1	1	1
	O3		<del>50</del>			<del>500</del> 300	1	1	1	0
Demand:		<del>250</del>	<del>350</del>	400	<del>200</del>	1200				
Column Difference:		1	2	2	2					
		-	2	2	2					
		-	3	2	7					
		-	3	2	-					

- Now there is only one column so select the cell with the least cost and allocate the value.

		Destination				Supply	Row Difference			
		D1	D2	D3	D4					
Source	O1	<del>2</del>	<del>300</del>	<del>1</del>	<del>7</del>	<del>300</del> 0	2	3	-	-
	O2	<del>250</del>				400 150	3	1	1	1
	O3		<del>50</del>	<del>250</del>	<del>200</del>	<del>500</del> 300	1	1	1	0
Demand:		<del>250</del>	<del>350</del>	400	<del>200</del>	1200 0				
Column Difference:		1	2	2	2					
		-	2	2	2					
		-	3	2	7					
		-	3	2	-					

- Now there is only one cell so allocate the remaining demand or supply to the cell

		Destination				Supply	Row Difference			
		D1	D2	D3	D4					
Source	O1	<del>2</del>	300	<del>7</del>	<del>4</del>	<del>300</del> 0	2	3	-	-
	O2	250	<del>6</del>	150	<del>9</del>	<del>400</del> 150 0	3	1	1	1
	O3	<del>8</del>	50	<del>3</del>	250	<del>500</del> 300 200	1	1	1	0
Demand:		<del>250</del> 0	<del>350</del> 50 0	<del>400</del> 150 0	<del>200</del> 0	1200 0				
Column Difference:		1	2	2	2					
		-	2	2	2					
		-	3	2	7					
		-	3	2	-					

- No balance remains. So multiply the allocated value of the cells with their corresponding cell cost and add all to get the final cost i.e.  $(300 * 1) + (250 * 2) + (50 * 3) + (250 * 3) + (200 * 2) + (150 * 5) = 2850$

**d) Transportation Problem: Unbalanced problem**

In this session, the method to solve the unbalanced transportation problem will be discussed.

Below transportation problem is an unbalanced transportation problem.

		Destination					Supply(S <sub>i</sub> )
		D1	D2	D3	D4	D5	
Source	O1	5	1	8	7	5	15
	O2	3	9	6	7	8	25
	O3	4	2	7	6	5	42
	O4	7	11	10	4	9	35
Demand(d <sub>j</sub> ):		30	20	15	10	20	117
							95

The problem is unbalanced because the sum of all the supplies i.e. **O1, O2, O3** and **O4** is not equal to the sum of all the demands i.e. **D1, D2, D3, D4** and **D5**.

**Solution:**

In this type of problem, the concept of a dummy row or a dummy column will be used. As in this case, since the supply is more than the demand so a dummy demand column will be added and a demand of (total supply – total demand) will be given to that column i.e.  $117 - 95 = 22$  as shown in the image below. If demand were more than the supply then a dummy supply row would have been added.

		Destination					Supply(S <sub>i</sub> )	
		D1	D2	D3	D4	D5		
Source	O1	5	1	8	7	5	0	15
	O2	3	9	6	7	8	0	25
	O3	4	2	7	6	5	0	42
	O4	7	11	10	4	9	0	35
Demand(d <sub>j</sub> ):		30	20	15	10	20	22	117
							117	

Now that the problem has been updated to a balanced transportation problem, it can be solved using any one of the following methods to solve a balanced transportation problem as discussed in the earlier posts:

1. NorthWest Corner Method
2. Least Cost Cell Method
3. Vogel's Approximation Method

# Optimal solution: MODI Method – UV Method

There are two phases to solve the transportation problem. In the first phase, the initial basic feasible solution has to be found and the second phase involves optimization of the initial basic feasible solution that was obtained in the first phase. There are three methods for finding an initial basic feasible solution,

1. NorthWest Corner Method
2. Least Cost Cell Method
3. Vogel's Approximation Method

Will discuss how to optimize the initial basic feasible solution through an explained example. Consider the below transportation problem.

		Destination				Supply( $S_i$ )
		D1	D2	D3	D4	
Source	O1	3	1	7	4	250
	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand( $D_j$ ):		200	300	350	150	

## Solution:

**Step 1:** Check whether the problem is balanced or not.

If the total sum of all the supply from sources **O1**, **O2**, and **O3** is equal to the total sum of all the demands for destinations **D1**, **D2**, **D3** and **D4** then the transportation problem is a balanced transportation problem.

		Destination				Supply( $S_i$ )
		D1	D2	D3	D4	
Source	O1	3	1	7	4	250
	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand( $D_j$ ):		200	300	350	150	1000

**Note:** If the problem is not unbalanced then the concept of a dummy row or a dummy column to transform the unbalanced problem to balanced can be followed as discussed.

**Step 2:** Finding the initial basic feasible solution.

Any of the three aforementioned methods can be used to find the initial basic feasible solution. Here, NorthWest Corner Method will be used. And according to the NorthWest Corner Method this is the final initial basic feasible solution:

		Destination				
		D1	D2	D3	D4	Supply(S <sub>i</sub> )
Source	O1	200	50			<del>250</del> 50 0
	O2		250	100		<del>350</del> 100 0
	O3			250	150	<del>400</del> 150 0
Demand(D <sub>j</sub> ):		<del>200</del> 0	<del>300</del> 250 0	<del>350</del> 250 0	<del>150</del> 0	1000

Now, the total cost of transportation will be  $(200 * 3) + (50 * 1) + (250 * 6) + (100 * 5) + (250 * 3) + (150 * 2) = 3700$ .

**Step 3:** U-V method to optimize the initial basic feasible solution.

The following is the initial basic feasible solution:

200	50		
3	1	7	4
	250	100	
2	6	5	9
8	3	250	150
		3	2

– For U-V method the values  $u_i$  and  $v_j$  have to be found for the rows and the columns respectively. As there are three rows so three  $u_i$  values have to be found i.e.  $u_1$  for the first row,  $u_2$  for the second row and  $u_3$  for the third row.

Similarly, for four columns four  $v_j$  values have to be found i.e.  $v_1, v_2, v_3$  and  $v_4$ . Check the image below:

	$v_1=$	$v_2=$	$v_3=$	$v_4=$
$u_1=$	200 3	50 1	7	4
$u_2=$	2	250 6	100 5	9
$u_3=$	8	3	250 3	150 2

There is a separate formula to find  $u_i$  and  $v_j$ ,

$u_i + v_j = C_{ij}$  where  $C_{ij}$  is the cost value only for the allocated cell.

Before applying the above formula we need to check whether  $m + n - 1$  is equal to the total number of allocated cells or not where  $m$  is the total number of rows and  $n$  is the total number of columns.

In this case  $m = 3, n = 4$  and total number of allocated cells is 6 so  $m + n - 1 = 6$ . The case when  $m + n - 1$  is not equal to the total number of allocated cells will be discussed in the later posts.

Now to find the value for  $u$  and  $v$  we assign any of the three  $u$  or any of the four  $v$  as 0. Let we assign  $u_1 = 0$  in this case. Then using the above formula we will get  $v_1 = 3$  as  $u_1 + v_1 = 3$  (i.e.  $C_{11}$ ) and  $v_2 = 1$  as  $u_1 + v_2 = 1$  (i.e.  $C_{12}$ ). Similarly, we have got the value for  $v_2 = 3$  so we get the value for  $u_2 = 5$  which implies  $v_3 = 0$ . From the value of  $v_3 = 0$  we get  $u_3 = 3$  which implies  $v_4 = -1$ .

	$v_1=-3$	$v_2=1$	$v_3=0$	$v_4=-1$
$u_1=0$	3	250 1	7	4
$u_2=5$	200 2	50 6	100 5	9
$u_3=3$	8	3	250 3	150 2

Now, compute penalties using the formula  $P_{ij} = u_i + v_j - C_{ij}$  only for unallocated cells. We have two unallocated cells in the first row, two in the second row and two in the third row. Let's compute this one by one.

1. For  $C_{13}$ ,  $P_{13} = 0 + 0 - 7 = -7$  (here  $C_{13} = 7$ ,  $u_1 = 0$  and  $v_3 = 0$ )
2. For  $C_{14}$ ,  $P_{14} = 0 + (-1) - 4 = -5$
3. For  $C_{21}$ ,  $P_{21} = 5 + 3 - 2 = 6$
4. For  $C_{24}$ ,  $P_{24} = 5 + (-1) - 9 = -5$
5. For  $C_{31}$ ,  $P_{31} = 3 + 3 - 8 = -2$
6. For  $C_{32}$ ,  $P_{32} = 3 + 1 - 3 = 1$

**The Rule:** If we get all the penalties value as zero or negative values that mean the optimality is reached and this answer is the final answer. But if we get any positive value means we need to proceed with the sum in the next step.

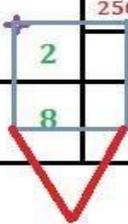
Now find the maximum positive penalty. Here the maximum value is 6 which corresponds to  $C_{21}$  cell. Now this cell is new basic cell. This cell will also be included in the solution.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	200 3	50 1	7 7	4 4
$u_2 = 5$	2 +	250 6	100 5	9 9
$u_3 = 3$	8 8	3 3	250 3	150 2

**The rule for drawing closed-path or loop.** Starting from the new basic cell draw a closed-path in such a way that the right angle turn is done only at the allocated cell or at the new basic cell.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	200 3	50 1	7 7	4 4
$u_2 = 5$	2 +	250 6	100 5	9 9
$u_3 = 3$	8 8	3 3	250 3	150 2





**Right angle turn is not permitted in unallocated cell.**

$v_1=3$     $v_2=1$     $v_3=0$     $v_4=-1$

$u_1 = 0$	200	50		
	3	1	7	4
$u_2 = 5$	2	6	5	9
$u_3 = 3$	8	3	3	2

✓

Assign alternate plus-minus sign to all the cells with right angle turn (or the corner) in the loop with plus sign assigned at the new basic cell.

$v_1=3$     $v_2=1$     $v_3=0$     $v_4=-1$

$u_1 = 0$	200	50		
	-	3	1	+
$u_2 = 5$	+	2	6	-
$u_3 = 3$	8	3	3	2

Consider the cells with a negative sign. Compare the allocated value (i.e. 200 and 250 in this case) and select the minimum (i.e. select 200 in this case). Now subtract 200 from the cells with a minus sign and add 200 to the cells with a plus sign. And draw a new iteration. The work of the loop is over and the new solution looks as shown below.

		250		
	3	1	7	4
200	2	50	100	9
	8	3	3	2

Check the total number of allocated cells is equal to  $(m + n - 1)$ . Again find  $u$  values and  $v$  values using the formula  $u_i + v_j = C_{ij}$  where  $C_{ij}$  is the cost value only for allocated cell. Assign  $u_1 = 0$  then we get  $v_2 = 1$ . Similarly, we will get following values for  $u_i$  and  $v_j$ .

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	3	250 1	7	4
$u_2 = 5$	200 2	50 6	100 5	9
$u_3 = 0$	8	3	250 3	150 2

Find the penalties for all the unallocated cells using the formula  $P_{ij} = u_i + v_j - C_{ij}$ .

1. For  $C_{11}$ ,  $P_{11} = 0 + (-3) - 3 = -6$
2. For  $C_{13}$ ,  $P_{13} = 0 + 0 - 7 = -7$
3. For  $C_{14}$ ,  $P_{14} = 0 + (-1) - 4 = -5$
4. For  $C_{24}$ ,  $P_{24} = 5 + (-1) - 9 = -5$
5. For  $C_{31}$ ,  $P_{31} = 0 + (-3) - 8 = -11$
6. For  $C_{32}$ ,  $P_{32} = 3 + 1 - 3 = 1$

There is one positive value i.e. 1 for  $C_{32}$ . Now this cell becomes new basic cell.

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	3	250 1	7	4
$u_2 = 5$	200 2	50 6	100 5	9
$u_3 = 3$	8	3 +	250 3	150 2

Now draw a loop starting from the new basic cell. Assign alternate plus and minus sign with new basic cell assigned as a plus sign.

	$v_1=-3$	$v_2=1$	$v_3=0$	$v_4=-1$
$u_1=0$	3	250 1	7	4
$u_2=5$	200 2	50 -	100 6 +5	9
$u_3=3$	8	+ 3	250 -	150 3 2

Select the minimum value from allocated values to the cell with a minus sign. Subtract this value from the cell with a minus sign and add to the cell with a plus sign. Now the solution looks as shown in the image below:

		250		
	3	1	7	4
200			150	
2	6	5	9	
8	50	3	200	150
		3		2

Check if the total number of allocated cells is equal to  $(m + n - 1)$ . Find  $u$  and  $v$  values as above.

	$v_1=-2$	$v_2=1$	$v_3=1$	$v_4=0$
$u_1=0$	3	250 1	7	4
$u_2=4$	200 2		150 5	9
$u_3=2$	8	50 3	200 3	150 2

Now again find the penalties for the unallocated cells as above.

1. For  $P_{11} = 0 + (-2) - 3 = -5$

2. For  $P_{13} = 0 + 1 - 7 = -6$

3. For  $P_{14} = 0 + 0 - 4 = -4$

4. For  $P_{22} = 4 + 1 - 6 = -1$

5. For  $P_{24} = 4 + 0 - 9 = -5$

6. For  $P_{31} = 2 + (-2) - 8 = -8$

All the penalty values are negative values. So the optimality is reached.

Now, find the total cost i.e.  $(250 * 1) + (200 * 2) + (150 * 5) + (50 * 3) + (200 * 3) + (150 * 2) =$   
**2450**

## Transportation Problem: Degeneracy in Transportation Problem

This session will discuss degeneracy in transportation problem through an explained example.

		Destination					
		D1	D2	D3	D4	D5	Supply(S <sub>i</sub> )
Source	S1	10	2	3	15	9	35
	S2	5	10	15	2	4	40
	S3	15	5	14	7	15	20
	S4	20	15	13	25	8	30
Demand(D <sub>j</sub> ):		20	20	40	10	35	

### Solution:

This problem is balanced transportation problem as total supply is equal to total demand.

		Destination					
		D1	D2	D3	D4	D5	Supply(S <sub>i</sub> )
Source	S1	10	2	3	15	9	35
	S2	5	10	15	2	4	40
	S3	15	5	14	7	15	20
	S4	20	15	13	25	8	30
Demand(D <sub>j</sub> ):		20	20	40	10	35	125

### Initial basic feasible solution:

Least Cost Cell Method will be used here to find the initial basic feasible solution. One can also use NorthWest Corner Method or Vogel's Approximation Method to find the initial basic feasible solution.

Using Least Cost Cell Method we get the following solution.

		Destination					Supply(S <sub>i</sub> )
		D1	D2	D3	D4	D5	
Source	S1	10	20 2	15 3	15	9	35
	S2	5	10	15	10 2	30 4	40
	S3	20 15	5	14	7	15	20
	S4	20	15	25 13	25	5 8	30
Demand(D <sub>j</sub> ):		20	20	40	10	35	125

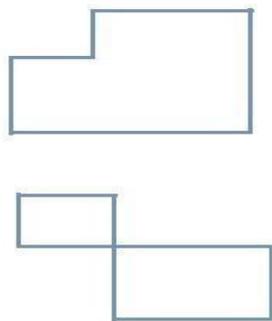
### Optimization of the solution using U-V Method:

Check whether  $m + n - 1 =$  total number of allocated cells. In this case  $m + n - 1 = 4 + 5 - 1 = 8$  where as total number of allocated cells are 7, hence this is the case of degeneracy in transportation problem. So in this case we convert the necessary number (in this case it is  $m + n - 1 -$  total number of allocated cells i.e.  $8 - 7 = 1$ ) of unallocated cells into allocated cells to satisfy the above condition.

#### Steps to convert unallocated cells into allocated cells:

- Start from the least value of the unallocated cell.
- Check the loop formation one by one.
- There should be no closed-loop formation.
- Select that loop as a new allocated cell and assign a value 'e'.

The closed loop can be in any form but all the turning point should be only at allocated cell or at the cell from the loop is started.



There are 13 unallocated cells. Select the least value (i.e. 5 in this case) from unallocated cells. There are two 5s here so you can select randomly any one. Lets select the cell with star marked.

		Destination					
		D1	D2	D3	D4	D5	Supply(S <sub>i</sub> )
Source	S1	10	20 2	15 3	15	9	35
	S2	5	10	15	10 2	30 4	40
	S3	20 15	5	14	7	15	20
	S4	20	15	25 13	25	5 8	30
Demand(D <sub>j</sub> ):		20	20	40	10	35	125

Check if there is any closed-loop formation starting from this cell. If a closed-loop is drawn from this cell following the condition for closed-loop then it can be observed that this cell cannot be reached to complete the closed-loop. So this cell will be selected and assigned a random value 'e'.

		Destination					
		D1	D2	D3	D4	D5	Supply(S <sub>i</sub> )
Source	S1	10	20 2	15 3	15	9	35
	S2	5	10	15	10 2	30 4	40
	S3	20 15	5 e	14	7	15	20
	S4	20	15	25 13	25	5 8	30
Demand(D <sub>j</sub> ):		20	20	40	10	35	125

**Note:** If the closed loop would have been formed from that cell then we would try another cell with least value and do the same procedure and check whether closed loop is possible or not.

Now total number of allocated cells becomes 8 and  $m + n - 1 = 4 + 5 - 1 = 8$ . Now this solution can be optimized using U-V method. We get the below solution after performing optimization using U-V method.

	e	35		
10	2	3	15	9
20			10	10
5	10	15	2	4
15	20+e	5	14	7
		5		25
20	15	13	25	8

The presence of two 'e' in the final solution means after doing some iterations during optimization, the condition for degeneracy will be met once again.

While finding the total cost, just leave the 'e' and multiply the allocated value with its cell's cost value and add all of them. So, the transportation cost is  $(35 * 3) + (20 * 5) + (10 * 2) + (10 * 4) + (20 * 5) + (5 * 13) + (25 * 8) = 630$ .