FOURIER SERIES

1. A function f(x) can be expressed as a Fourier series in $(0, 2\pi)$ as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos nx+b_n \sin nx\right]$$

Where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

2. A function f(x) can be expressed as a Fourier series in $(-\pi, \pi)$ as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos nx+b_n \sin nx\right]$$

Where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

3. A function f(x) can be expressed as a Fourier series in (0, 2L) as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L}+b_n \sin\frac{n\pi x}{L}\right]$$

Where $a_0 = \frac{1}{L}\int_0^{2L} f(x)\,dx$

$$a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}\,dx$$

4. A function f(x) can be expressed as a Fourier series in (-L, L) as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right]$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

5. If a function f(x) is an even function in (-π, π) then its Fourier series expansion contains cosine terms only, i.e. $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos nx\right]$

Where $a_0=\frac{2}{\pi}\int_{0}^{\pi} f(x)\,dx$

$$a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx$$

6. If a function f(x) is an odd function in (-π, π) then its Fourier series expansion contains sine terms only, i.e. $f(x)=\sum_{n=1}^{\infty}\left[b_n\sin nx\right]$

$$b_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$$

7. If a function f(x) is an even function in (-L, L) then its Fourier series expansion contains cosine terms only, i.e. $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L}\right]$

$$a_0=\frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n=\frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

8. If a function f(x) is an odd function in (-L, L) then its Fourier series expansion contains sine terms only, i.e. $f(x)=\sum_{n=1}^{\infty}\left[b_n \sin\frac{n\pi x}{L}\right]$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

9. A function f(x) can be expressed as a half range Fourier cosine series in (0, π) as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos nx\right]$$

Where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$$

10. A function f(x) can be expressed as a half range Fourier sine series in (0, π) as

$$f(x)=\sum_{n=1}^{\infty}\left[b_n \sin nx\right]$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx dx$$

11. A function f(x) can be expressed as a half range Fourier cosine series in (0, L) as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L}\right]$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

12. A function f(x) can be expressed as a half range Fourier sine series in (0, L) as

$$f(x)=\sum_{n=1}^{\infty}\left[b_n \sin\frac{n\pi x}{L}\right]$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

13. Parseval's identity in (0, 2π)

$$\int_0^{2\pi} |f(x)|^2 dx = \pi\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2} + b_n^{\,2}\right]\right]$$

14. Parseval's identity in (-π, π)

$$\int_{-\pi}^{\pi} |f(x)|^2 dx = \pi\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2} + b_n^{\,2}\right]\right]$$

15. Parseval's identity in (0, 2L)

$$\int_0^{2L} |f(x)|^2 dx = L\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2} + b_n^{\,2}\right]\right]$$

16. Parseval's identity in (-L, L)

$$\int_{-L}^{L} |f(x)|^2 dx = L\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2} + b_n^{\,2}\right]\right]$$

17. Parseval's identity in (0, π) provided half range cosine series

$$\int_0^{\pi} |f(x)|^2 dx = \frac{\pi}{2}\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2}\right]\right]$$

18. Parseval's identity in (0, π) provided half range sine series

$$\int_0^{\pi} |f(x)|^2 dx = \frac{\pi}{2}\left[\sum_{n=1}^{\infty}\left[b_n^{\,2}\right]\right]$$

19. Parseval's identity in (0, L) provided half range cosine series

$$\int_0^{L} |f(x)|^2 dx = \frac{L}{2}\left[\frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty}\left[a_n^{\,2}\right]\right]$$

20. Parseval's identity in (0, L) provided half range sine series

$$\int_0^{L} |f(x)|^2 dx = \frac{L}{2}\left[\sum_{n=1}^{\infty}\left[b_n^{\,2}\right]\right]$$